

# Errata for *Conquering the Physics GRE*, edition 1

August 31, 2014

This document contains corrections to *Conquering the Physics GRE*. A word about versioning. Each edition has different sub-versions. So there is a version 1.0 and a version 2.0 for the first edition. Right now we are on version 1.0 of the second edition. Next to each item is a number indicating the most recent version in which the error exists (e.g. 1.4 means that the error exists in version 1.4, but was fixed in version 1.5), and the page number and section of the error in this version. You can find the version number of your edition in the front matter of your book, on the first page of your chapter or exam, or in the filename of your chapter or exam for version 1.0 documents. Our numbering system is as follows: the first number indicates a new printing of the book available on Amazon, and the second number indicates minor revisions which are available as PDF's only.

Note that because the exams were released before the book as PDFs, version 1.0 of the book actually contains versions 1.2.0, 2.2.0, and 3.2.0 of the exams. In order to clarify this, errata for the exams list both the relevant version number for the book and for the exam, with the book version number in parentheses and preceded by a “b”. For example, 1.1.2 (b:1.0) means that the latest version containing the error is version 1.1.2 of the exam PDFs, and version 1.0 of the book. Since the vast majority of exams were purchased since the release of the book, corrections to older versions of the exams are listed in a separate document.

Finally, note that both page numbers and section numbers may change slightly between each version of the book, though section numbers should be more stable. The page numbers listed here refer to the page of the error in the *latest version in which the error is still present*.

Feel free to contact us at [physics@physicsgreprep.com](mailto:physics@physicsgreprep.com) if any of the information here is unclear.

## Resources

- p. xii: the Specialized Topics section should refer to Ch. 1 of Griffiths' book (though much of the rest of the book is good too!).

# 1 Classical Mechanics

- 2.0 - §1.1.3 p. 4: The fourth paragraph refers to Newton’s first law, when it should refer to Newton’s *third* law. The same holds for several similar occurrences throughout the chapter.
- 2.0 - §1.1.4 p. 5: In problem 3, none of the answer choices are correct as written. Choice D should be changed to read:

$$\left(\mu_1 + \frac{1}{\mu_2}\right)(m + M)g.$$

- 2.0 - §1.4.5 p. 21: Problem 4 should not assume negligible loss of energy to friction, since this is what causes the transfer of angular momentum in the first place. The last sentence should simply read: “What is  $\omega$ ?”
- 2.0 - §1.6.1 p. 28-29: The derivation of the effective potential from the Lagrangian contains a subtle spurious factor of  $-1$ . The section should be rewritten as follows to derive the effective potential from the equations of motion:

The radial behavior of the orbit is of course described by the Euler-Lagrange equation for the radial coordinate

$$\frac{d}{dt}(m\dot{r}) = mr\dot{\phi}^2 - U'(r).$$

Substituting for  $\dot{\phi}$  in terms of  $l$ , we get

$$m\ddot{r} = \frac{l^2}{mr^3} - U'(r).$$

First of all, we have reduced a complex system of partial differential equations in three dimensions to a single ordinary differential equation, which we may have some hope of understanding. And secondly, this looks suspiciously like Newton’s second law of motion. We can improve the resemblance by “factoring” the derivative on the right hand side to find

$$m\ddot{r} = -\frac{d}{dr}\left(\frac{l^2}{2mr^2} + U\right).$$

This is now in exactly the same form as Newton’s second law, except that the “potential” now has an additional term depending on  $l$ . We call the expression in parentheses the *effective potential*,

$$V(r) = \frac{l^2}{2mr^2} + U(r).$$

- 2.0 - §1.6.4 p. 32: Problem 3 should specify that  $m$  is negligibly small compared to  $M$ , and choice E should be  $GM/2v^2$ .

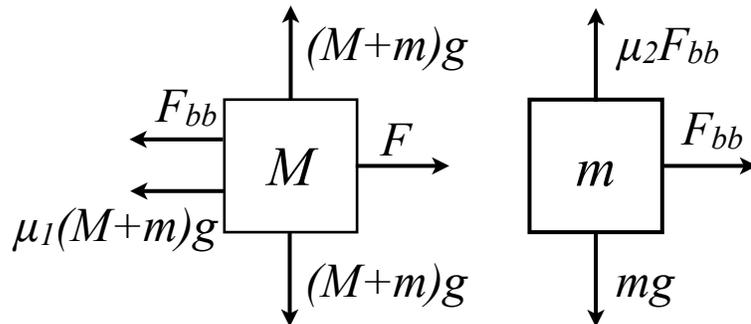
- 2.0 - §1.7.1 p. 34: The pair of equations below equation (1.45) should be replaced by

$$\begin{aligned} m\omega^2 a_1 &= 2ka_1 - ka_2 \\ m\omega^2 a_2 &= 2ka_2 - ka_1. \end{aligned}$$

- 2.0 - §1.7.2 p. 36: The equation immediately preceding (1.49) is missing a factor of two and should read

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = F_0 \cos \omega t.$$

- 2.0 - §1.7.4 p. 40: The last sentence of problem 2 should read “What horizontal distance  $x$  does the ball travel before returning to its height at launch?”
- 2.0 - §1.8.1 p. 41: The sentence fragment before equation (1.55) should be deleted.
- 2.0 - §1.8.1 p. 41: The last term on the right-hand side of the last equation on the page should be  $p_1$ , NOT  $p_2$ .
- 2.0 - §1.9 p. 44: The free body diagram in the problem should have every instance of  $Mg$  changed to  $(M + m)g$  because the block of mass  $M$  supports the weight of both blocks as long as mass  $m$  is held in place by friction. The corrected diagram should look like:



The last sentence and equation should be corrected to read:

The net force on the first block is  $F - F_{bb} - \mu_1(m + M)g = F - mg/\mu_2 - \mu_1(m + M)g$ , so we set its acceleration equal to  $g/\mu_2$  and solve:

$$\frac{F - mg/\mu_2 - \mu_1(m + M)g}{M} = \frac{g}{\mu_2} \implies F = \left( \mu_1 + \frac{1}{\mu_2} \right) (m + M)g.$$

The solution to this problem should remain D, as the answer choices were changed to include the correct solution.

- 2.0 - §1.9 p. 48: The first line of the equation beginning  $\Delta E...$  has  $R$  and  $r$  interchanged. The following equations are still correct. The first line should instead read:  $\Delta E = \frac{1}{2}M (r^2\omega^2 - R^2\omega_0^2)$ .

- 2.0 - §1.9 p. 48: The expression for  $z$  should not have a minus sign.
- 2.0 - §1.9 p. 53: The final sentence should say “Solving for  $v_2...$ ” not “Solving for  $v_1...$ ”

## 2 Electricity and Magnetism

- 2.0 - §2.1.8 p. 67: Problem 3 should read “ $d \ll a$ ” NOT “ $d \gg a$ ”.
- 2.0 - §2.2.7 p. 74: The last sentence of problem 2 should read “which best describes the direction of the force from the top loop on electrons in the bottom loop?”
- 2.0 - §2.3.4 p. 79. The mass of the rod in problem 3 should be given as  $m$  in the problem statement.
- 2.0 - §2.4.4 p. 82: The answer choices for C, D, and E should have  $a$  instead of  $A$ .
- 2.0 - §2.6.1 p. 85. The sentence before equation (2.62) should say that the time average of the Poynting vector is the average power per unit area, NOT per unit time.
- 2.0 - §2.8 p. 97: In the solution to circuits problem 1, the expression in the second line should read  $(1/R + 2/R)^{-1}$  (the exponent  $-1$  is missing).
- 2.0 - §2.8 p. 97: In the solution to circuits problem 2, the second to last line of the final equation array should have a factor of  $1/2$  on the LHS, rather than a factor of 2. The solution of the problem is still correct.

## 3 Optics and Waves

- 2.0 - §3.1.3 p. 101: last sentence should read “...phase velocity can in general be a function of  $k...$ ”, rather than “...group velocity...” (group velocity can also be a function of  $k$ , but this particular statement applies more correctly for phase velocity)
- 2.0 - §3.1.6 p. 103: There should not be a hat on the  $\mathbf{r}$  in the first equation of this section.
- 2.0 - §3.2 p. 104: The sixth sentence of this section is missing the word “same”. It should read “Now, another wave  $g(x) = B \cos(kx + \delta_2)$  with the same wavenumber but a different phase, will interfere with  $f$ .”
- 2.0 - §3.2.1 p. 106: The diagram should say  $\Delta x = d \sin \theta$ .
- 2.0 - §3.3.2 p. 111: The sentence immediately after equation (3.24), says that the sign convention for focal lengths of lenses is the same as for mirrors. It should state the convention is the *opposite* as for mirrors.

- 2.0 - §3.5 p. 118: problem 14 should specify “the image(s) of the object formed *by the lens* in this configuration.”
- 2.0 - §3.6 p. 120: The solution to problem 8 should refer to “the old second maximum *after the central maximum...*” and the “new *third* minimum.”

## 4 Thermodynamics and Statistical Mechanics

- 2.0 - §4.1.2 p. 126: eq. (4.8) should have  $\delta Q$  in the integral, not  $dQ$ .
- 2.0 - §4.1.3 p. 128: in the second paragraph, all occurrences of “heat capacity” should read “internal energy.”
- 2.0 - §4.2.4 p. 131: eq. (4.30) should have  $\delta Q$  on the LHS, not  $dQ$ .
- 2.0 - §4.2.6 p. 134: The last paragraph of the page should refer to an increase in pressure at constant *volume*, NOT at constant temperature.
- 2.0 - §4.2.6 p. 135: the prefactor on the RHS of the equation immediately above eq. (4.41) should be  $\frac{V^N}{N!}$ , NOT  $\frac{V}{N!}$ .
- 2.0 - §4.4 p. 138: problem 3 should read “at very high temperatures,” NOT “at room temperature.”
- 2.0 - §4.4 p. 140: problem 9 should specify that steps 12 and 34 are reversible adiabatic, not just adiabatic.
- 2.0 - §4.4 p. 141: problem 13 be changed to the following for dimensional consistency:  
A grand canonical ensemble of fermions at chemical potential  $\mu$  has a density of states given by  $\rho(\epsilon) = Ae^{-\kappa\epsilon}$ . At zero temperature, what is the average number of particles in the system?

(A) 1

(B)  $\frac{A}{\kappa}e^{-\kappa\mu}$

(C)  $\frac{A}{\kappa}(1 - e^{-\kappa\mu})$

(D)  $\frac{A}{\kappa} - e^{-\kappa\mu}$

(E) 0

The solution should be changed accordingly.

- 2.0 - §4.4 p. 140: problem 12 should read “what is the value of the Bose-Einstein distribution function,” NOT “what is the average occupation number of the system.” Since any physical system has a finite number of particles, the actual occupation number can never be infinite.

## 5 Quantum Mechanics and Atomic Physics

- 2.0 - §5.1.2 p. 149: equation (5.9) should read  $\langle a|\hat{A}b\rangle := \langle \hat{A}^\dagger a|b\rangle$ . There should be NO dagger on the bracket on the LHS.
- 2.0 - §5.1.2 p. 149: the first sentence of the paragraph below equation (5.9) should read “... then both sides of (5.9) contain the same operator  $A$ ,” NOT “... then both sides of (5.9) are the same.”
- 2.0 - §5.1.3 p. 150: paragraph below (5.16),  $\psi(n)$  should read  $\psi_n(x)$ .
- 2.0 - §5.1.3 p. 151: third bullet point, there are several factors of  $\hbar$  missing. All the exponentials should read  $e^{+iE_n t/\hbar}$  and so on, and the last sentence should have  $\cos((E_2 - E_1)t/\hbar)$ .
- 2.0 - §5.1.5 p. 154: In problem 2, the wavefunction should be  $\Psi(x) = Ae^{-x^2/2\ell^2}$  for dimensional consistency, all instances of  $x$  in the answer choices should be replaced by  $x/\ell$ , and all instances of  $\pi^{-1/4}$  should be replaced by  $A$ . The solution should be changed accordingly.
- 2.0 - §5.1.5 p. 157: the mass values given in problem 10 are incorrect, and for clarity should also be given in units of  $\text{MeV}/c^2$ . The first two sentences should read: “An unstable particle with a lifetime of  $1.0 \times 10^{-23}$  s and a mass of  $500 \text{ MeV}/c^2$  is measured in a new experiment to have a mass of  $450 \text{ MeV}/c^2$ . The mass resolution of the experiment is  $10 \text{ MeV}/c^2$ .” The corresponding changes should be made to the solutions on p. 197.
- 2.0 - §5.3.3 p. 164: The dimensions of the constant  $A$  are stated incorrectly in the second paragraph of this section. They are “energy \* length”.
- 2.0 - §5.4 p. 168: in the first paragraph, the partial derivatives should read  $\partial/\partial x$  instead of  $\partial/\partial_x$ , etc.
- 2.0 - §5.4.3 p. 173: last sentence of the first paragraph should read “stronger by a factor of 2” NOT “stronger by a factor of 4.”
- 2.0 - §5.5 p. 175: The second paragraph of this section should say that a spin-1/2 particle has states with  $m_s = +1/2$  and  $m_s = -1/2$ , NOT  $m_s = \pm 1$ .
- 2.0 - §5.5.3 p. 179: The third equation from the bottom of the page is missing a factor of  $\hbar$  from the RHS: it should read  $\hbar(|\downarrow\rangle|\uparrow\rangle + |\uparrow\rangle|\downarrow\rangle)$ .

## 6 Special Relativity

- 2.0 - §6.6 p. 217: Problem 1 should specify that  $\bar{S}$  is traveling in the  $\hat{x}$  direction.
- 2.0 - §6.7 p. 221: In the solution to problem 7, the quantity  $\Delta x$  is defined incorrectly. It should be defined as the distance traveled by the Romulan warship while the photon torpedo is in transit. Moreover, the quantity  $x_0$  is not defined. It should be defined as the distance between the Enterprise and the warship in the frame of the Enterprise.

## 7 Laboratory Methods

- 2.0 - §7.6 p. 237: The last sentence of problem 2 should read “If A and B are independent events, the probability distribution for the *sum* of A and B...”
- 2.0 - §7.6 p. 239: the circuit diagram in problem 7 should have a resistor between the (positive side of the) voltage source and the diode.
- 2.0 - §7.7 p. 241: the solution to problem 7 should read “the diode is reverse biased and effectively an open circuit” (NOT “a short”)

## 8 Specialized Topics

## 9 Special Tips and Tricks for the Physics GRE

- 2.0 - §9.3 p. 264: the equations for  $\tau_g$  and  $\tau_F$  are both missing a factor of  $R$ .
- 2.0 - §9.6 p. 269: apparently we still can't do arithmetic. The RHS of the second-to-last equation on the page should be  $8 \times 10^7 \text{ W m}^{-2}$ , NOT  $8 \times 10^6$ . This missing factor of 10 carries through the calculation.

## 10 Exams

A number of errors were present in version 2.0 which made certain exam problems incorrect or confusing enough to warrant replacement. In using the practice exams, in version 2.0 or previous, please disregard the following problems and their solutions:

**Exam 2:** 46, 79, 92

**Exam 3:** 70

The specific issues with these problems are addressed in the errata below.

## 10.1 Exam 1

- b:2.0 - p. 284: The statement of problem 49 should say “heated from temperature  $T$  to temperature  $2T$  reversibly at constant volume” NOT “at constant temperature”.
- b:2.0 - p. 285: The statement of problem 51 should say “necessarily violate” instead of just “violate”. Choice C should also read “spin angular momentum” instead of just “angular momentum”.
- b:2.0 - p. 286: The statement of problem 56 should clarify that the spaceship velocity  $v$  is non-relativistic, and the speed  $v/2$  of the ejected fuel is in the frame of the spaceship.
- b:2.0 - p. 291: In the statement of problem 82, “Hamiltonian” should be replaced by “total energy”.
- b:2.0 - p. 364: The denominator of the last line of this problem states that  $4 \times 40 = 180$ . Indeed,  $4 \times 40 = 160$ , though the final answer is still correct. The corrected line should read:

$$r = \frac{\hbar c}{4 \times 40 \text{MeV}} = \frac{197}{160} \text{ fm} \simeq 1.23 \text{ fm}.$$

- b:2.0 - p. 371: The solution of problem 82 should begin with “Recalling that the total energy is  $H = T + U...$ ”
- b:2.0 - p. 372: The third sentence of the solution of problem 83 should end “changes from  $\infty$  to 0 on the interval  $[a, 2a]$ ,” NOT  $[0, 2a]$ .

## 10.2 Exam 2

- b:2.0 - p. 307: Problem 39 should read “real, orthonormal energy eigenfunctions” (the word “real” was omitted).
- b:2.0 - p. 309: there are several issues with problem 46, most importantly the fact that the transfer matrices of the beam splitter and one-way mirror were not defined. As this falls far outside the scope of the GRE, problem 46 will be replaced in the next edition of the book.
- b:2.0 - p. 311: problem 56 should specify that the spring is massless.
- b:2.0 - p. 311: the wavefunction is normalized incorrectly. The problem should read:  
A particle’s normalized spin wavefunction has the form

$$\psi(\theta, \phi) = \sqrt{\frac{3}{2\pi}} \sin \theta \cos 2\phi \sin \phi$$

What is the expectation value of the particle’s  $z$ -component of orbital angular momentum  $L_z$ ?

- (A) 0
- (B)  $-3\hbar/2\pi$
- (C)  $3\hbar/2\pi$
- (D)  $-3\hbar/\pi$
- (E)  $3\hbar/\pi$

- b:2.0 - p. 318: none of the answer choices to problem 79 are correct as stated. Despite appearances, the true EMF is continuous because of a cancellation between the magnetic current term which must be added to Faraday's Law in the presence of magnetic monopoles, and the ordinary flux term, taking into account the back EMF of a loop with nonzero inductance. This calculation is outside the scope of a GRE problem; problem 79 will be replaced in the next edition of this book.
- b:2.0 - p. 320: problem 92 is ill-defined because the source of the current which feeds the increasing line charge is never specified, and thus there is no unique solution to Maxwell's equations with the information given. This problem will be replaced in the next edition of the book.
- b:2.0 - p. 348: In the answer key, the answer to problem 5 is B NOT A, while the answer to problem 6 is A NOT B.
- b:2.0 - p. 376: in the solution to problem 5,  $\hbar c \simeq 200$  MeV/fm should read  $\hbar c \simeq 200$  MeV·fm, and  $\Delta E_0$  should read  $E_0$ .
- b:2.0 - p. 379: in the solution to problem 20,  $f \approx 0.11$  m should read  $f \approx -0.11$  m.
- b:2.0 - p. 382: as stated, the answer to problem 39 should be C, but with the corrected wording of the question, the answer remains E.
- b:2.0 - p. 382: in the solution to problem 45, the integral for  $V$  should read  $-\int_a^r E(r) dr$ , not  $-\oint_a^r E(r) dr$ .
- b:2.0 - p. 388: The answer to problem 70 should be B NOT A. The end of the third sentence should read "it acts as a virtual object for B," NOT "virtual image." The last sentence should read "Since  $s'_1 > 0$ , the first magnification is  $m_1 = -s'_1/s_1 < 0$ , so the first image is inverted. But both the second object and image distances are  $s_2$  and  $s'_2$  are negative, so  $m_2 < 0$ , and the final image is upright."
- b:2.0 - p. 388: in the solution to problem 71,  $\epsilon$  should be  $\epsilon_0$ .
- b:2.0 - p. 392: in the solution to problem 89,  $\delta l$  should read  $\Delta l$ , and the last sentence should read "3s must decay to 2p *in the electric dipole approximation.*"
- b:2.0 - p. 393: in the solution to problem 91, the first expression on the last line should read  $\sqrt{N}/N$ , not  $N/\sqrt{N}$ .

### 10.3 Exam 3

- b:2.0 - p. 327: Problem 10, the first three answer choices should specify that exactly one of each circuit element is present in the circuit. The question and corrected answer choices should read:

A circuit made only of which of the following circuit elements may function as a bandpass filter?

- (A) One resistor, one inductor, and one capacitor
  - (B) One resistor and one inductor
  - (C) One resistor and one capacitor
  - (D) Two resistors
  - (E) Two capacitors
- b:2.0 - p. 328: in problem 15, choice II,  ${}^6\text{Li}$  should read  ${}^7\text{Li}$ .
  - b:2.0 - p. 330: Problem 24 should specify that both loops are centered at the origin and that the torque is about the center of the smaller loop.
  - b:2.0 - p. 330: in problem 25, the last sentence should refer to “ratio of the acceleration,” not “ratio of the force.” (Fictitious forces are reference-frame dependent, but accelerations in a given reference frame are unambiguous.) The wording of the solution should be changed accordingly.
  - b:2.0 - p. 338: Problem 64 should specify that the string is fixed at both ends.
  - b:2.0 - p. 339: Depressingly, problem 70 does not make any sense at all. The problem should be reworded and the answers changed as shown below. The diagram is still correct for this reformulation.

“Two converging lenses of focal length  $f/2$  are placed in series, separated by a distance  $d$ . The object is placed a distance  $2f$  to the left of the left lens, and the image is located a distance  $f$  to the right of the right lens. What is  $d$ ?”

    - (A)  $(2/3)f$
    - (B)  $(5/3)f$
    - (C)  $(7/3)f$
    - (D)  $3f$
    - (E)  $9f$ ”
  - b:2.0 - p. 341: The last sentence of problem 80 should read: “What is the late-time amplitude of the spring oscillations, assuming friction is small but sufficient to damp out transient oscillations?”

- b:2.0 - p. 341: Problem 81 should specify that the origin of coordinates in both reference frames coincide.
- b:2.0 - p. 342: Choice B in problem 82 should be  $\sqrt{gh}$ , not  $\sqrt{gL}$ .
- b:2.0 - p. 342: Though it is correct as stated, the “not” in the first sentence of problem 84 should be capitalized (NOT) for added emphasis and consistency with GRE formatting standards.
- b:2.0 - p. 343: Problem 88 has two correct answers due to a typo: choice B should be  ${}^3\text{He}$ , not  ${}^3\text{H}$  (tritium is not produced to any appreciable extent in the  $pp$  cycle).
- b:2.0 - p. 345: Problem 100 should specify that  $v \ll c$ , and the last sentence should read “If the absorption peaks of two lines correspond to source velocities of 0 and  $v$ , what is the energy splitting between the lines to lowest order in  $v$ ?”
- b:2.0 - p. 350: The answer to problem 6 in the key should be C.
- b:2.0 - p. 398: the last line of the derivation in the solution to problem 5 should have a minus sign (not a plus sign) between the two terms in the integrand.
- b:2.0 - p. 399: None of the answers to problem 8 are correct as given. The trig identity given is incorrect and should read

$$\cos(45^\circ + \theta) = \cos 45^\circ \cos \theta - \sin 45^\circ \sin \theta,$$

resulting in a minus sign in the equation which follows,

$$g\theta = \Omega^2 R \left( \frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \theta \right).$$

The solution for  $\theta$  is then

$$\theta = \frac{\Omega^2 R}{2g + \Omega^2 R}.$$

- b:2.0 - p. 399: The second line of the solution to problem 9 has an error in the exponent of the denominator of the second fraction. This should be  $y^4$ , NOT  $y^2$ , so that the full line reads

$$= \frac{4x^2}{y^2}(\Delta x)^2 + \frac{x^4}{y^4}(\Delta y)^2. \quad (1)$$

The error does not propagate (no pun intended) and the final answer is correct.

- b:2.0 - p. 401: In the solution to problem 16, the third-to last equation should read  $p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$ . The error does not propagate and the rest of the solution is correct.

- b:2.0 - p. 402: In the solution to problem 20, the final expression for the energy should read  $4C^2v^4d^2T^3/3R$ . The solution and statement of the problem are otherwise correct.
- b:2.0 - p. 403: in the solution to problem 23, “half-open” should read “open”. The rest of the solution is correct.
- b:2.0 - p. 404: in the solution to problem 30, the first sentence should read “can be solved” and all instances of  $a$  should be  $d$ .
- b:2.0 - p. 407: In the solution to problem 40, the quadratic equation to solve has a sign flipped in the final term. It should read

$$0 = R^2 - Rr - r^2.$$

The error is not propagated and the rest of the solution, including the answer are correct.

- b:2.0 - p. 415: The solution to problem 70 makes even less sense than the nonsensical problem itself. The solution to the reformulated problem is as follows:

“B - This is a simple application of the thin lens equation. The first lens satisfies

$$\frac{2}{f} = \frac{1}{2f} + \frac{1}{q},$$

where  $q$  is some undetermined position of the image from the first lens. The second lens satisfies

$$\frac{2}{f} = \frac{1}{d - q} + \frac{1}{f}.$$

Solving for  $d$ , we obtain

$$d = \frac{5}{3}f.”$$

- b:2.0 - p. 415: in the solution to problem 73, the equation for the gauge transformation for  $V$  should read

$$V \rightarrow V - \frac{\partial f}{\partial t}.$$

- b:2.0 - p. 417: in the solution to problem 80, the second-to-last equation should have  $F_0/m$  on the right-hand side rather than  $F_0$ . The rest of the solution is correct.
- b:2.0 - p. 420: the first sentence of the solution to problem 94 should be reworded “We simply need to find the speed of the car at the top of the loop and then find what initial velocity is needed in order for gravity to provide the required centripetal force.”
- b:2.0 - p. 422: the solution to problem 98 is missing the constants  $A$ ,  $B$ , and  $C$ . The equations should read

$$\begin{aligned}\mathbf{F}(x, y, z) = -\nabla U &= -\hat{\mathbf{x}} \frac{\partial}{\partial x} (Ax) - \hat{\mathbf{y}} \frac{\partial}{\partial y} (By^2) + \hat{\mathbf{z}} \frac{\partial}{\partial z} (C \cos z) \\ &= -A\hat{\mathbf{x}} - 2By\hat{\mathbf{y}} - C \sin z\hat{\mathbf{z}}.\end{aligned}$$

- b:2.0 - p. 422: in problem 99, the final answer is correct, but the solution contains numerous errors. The solution should be rewritten as follows:

The ground state wavefunction of hydrogen is spherically symmetric, so we will denote it by  $\psi(r)$ . Defining the  $z$ -direction along the direction of the electric field, the perturbation Hamiltonian is  $\Delta H = eE_0z = eE_0r \cos \theta$ , and the first-order correction is given by

$$\int_0^\infty \int_0^{2\pi} \int_0^\pi \psi^*(r)\psi(r)(eE_0r \cos \theta)r^2 \sin \theta \, d\theta \, d\phi \, dr = 0.$$

However, the  $\theta$  integral vanishes:

$$\int_0^\pi \cos \theta \sin \theta \, d\theta = \frac{1}{2} \int_0^\pi \sin 2\theta \, d\theta = -\frac{1}{4} (\cos 2\pi - \cos 0) = 0.$$

Thus the first-order correction vanishes.

## 11 Errata for versions prior to edition 1, printing 2.0

### 11.1 General errata

- 1.1 - throughout the book we use “ln” and “log” interchangeably. However, the GRE reserves “ln” for the natural logarithm, so all natural logarithms are notated “ln” rather than “log” in printing 2.0 and on.

### 11.2 Classical Mechanics

- 1.1 - §1.1.3 p. 4: The last two sentences of section 1.1.4, beginning with “A *very* common mistake...”, are rather vague. While one must indeed be careful not to double-count the mass of the blocks, there is a simple way to solve this problem by treating the blocks as a single body. Replace these final two sentences with the following explanation: An alternative way to solve the problem is to note that the acceleration of the block-block system must satisfy  $F = (M + m)a$ . The block will begin to slip when the maximum static frictional force equals the actual frictional force of the lower block on upper block  $F_f = \mu mg = ma$ . Solving these two equations, we arrive at the same result  $\mu = F/((M + m)g)$ .
- 1.2 - §1.1.4 p. 5: In problem 1 of the section on blocks (in section 1.1.4), the coefficient of friction should be 0.5 NOT 0.6.

- 1.1 - §1.2 p. 7: The notation in the third equation on this page is ambiguous. In the example of the rocket, let  $u$  denote the vertical velocity of the rocket *instead* of  $v$ . This changes the second and third equations on p. 7 to  $v_f^y = \sqrt{u^2 + 2gh}$  and  $v_f = \sqrt{u^2 + w^2 + 2gh}$ , respectively.
- 1.1 - §1.2.1 p. 8: In the third sentence of the second-to-last paragraph of section 1.2.1, the words “is slowly” should be deleted.
- 1.0 - §1.2.2 p. 8: Choices (D) and (E) of problem 1 should read: “ $(1/2) \arcsin(gd/v^2)$ ” and “ $\arcsin(gd/v^2)$ ” respectively.
- 1.0 - §1.4.2 p. 18: The last sentence of section 1.4.2 should read: “The precession frequency is thus given by  $\omega_p = 2mgr/(m\omega R^2) = 2gr/\omega R^2$ .” The equation was misprinted.
- 1.0 - §1.4.5 p. 20: In the choices for problem 2, choice (A) should read “ $4\pi AR^6/3$ ”, and choice (B) should read “ $4\pi AR^6/9$ ”.
- 1.1 - §1.5.3 p. 25: The third sentence after the hamiltonian for free particle in polar coordinates in two dimensions is missing a verb. It should read: “As long as the potential is velocity- and time-independent...”
- 1.0 - §1.6.1 p. 28: The coordinates in the first paragraph of section 1.6.1 are mixed up. A revised version of the paragraph should read:

The fact that our potential has the form  $U(r)$  immediately gives us conservation laws which we can put right to use. First, let’s write down the Lagrangian for a particle of mass  $m$  moving in the potential  $U$ : after writing  $x$ ,  $y$ , and  $z$  in spherical coordinates, we find

$$L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}mr^2\sin^2\theta\dot{\phi}^2 - U(r).$$

(The polar angle  $\theta$  shows up in the kinetic energy roughly for the same reason that it shows up in the spherical volume element  $r^2 \sin \theta$ .) Reverting to classical reasoning for a bit, conservation of angular momentum implies the conservation of a whole *vector*  $\mathbf{L}$  (whose magnitude is  $l$ ), and the fact that the *direction* of this vector is constant means that *the particle is confined to a plane*. By spherical symmetry, we can choose this plane to be at  $\theta = \pi/2$ ; the second term (involving  $\dot{\theta}$ ) vanishes since  $\theta$  is constant, and  $\sin(\pi/2) = 1$  means the third term simplifies to  $\frac{1}{2}mr^2\dot{\phi}^2$ . Now, since  $U(r)$  is independent of the azimuthal angle  $\phi$ , so is the Lagrangian, and that gives us conservation of the conjugate momentum to  $\phi$ , which we identify as the ordinary angular momentum  $l$ :

$$l = mr^2\dot{\phi}.$$

- 1.0 - §1.7.1 p. 34: A subscript in equation 1.45 is misprinted. This equation should read:

$$m\ddot{x}_2 = -kx_2 - k(x_2 - x_1).$$

- 1.1 - §1.7.2 p. 35: In the last line, “This often be accomplished” should read “This can often be accomplished”
- 1.1 - §1.7.2 p. 36: The paragraph beginning with “The overdamped solution...” is a misleading explanation of overdamping. In fact, the damping in this case is so strong that no oscillation occurs. While the solutions to the underdamped case are sinusoidal function (i.e. *complex* exponentials), the solutions to the overdamped oscillator are *real* exponentials, with the position returning exponentially to its equilibrium position.
- 1.1 - §1.7.3 p. 37: The equation before equation 1.50 is missing a factor of  $L$  on the LHS. It should instead read:

$$mL\ddot{\theta} = -mg \sin \theta.$$

- 1.1 - §1.8.1 p. 41: The two equations immediately following equation (1.55) should contain factors of  $a^4$  and  $b^4$ , NOT  $a^2$  and  $b^2$ . The corrected equations should read:

$$\frac{v_1^2}{2} + \frac{p_1}{\rho} = \frac{v_1^2 a^4}{2b^4} + \frac{p_2}{\rho},$$

and

$$p_2 = \frac{\rho v_1^2}{2} \left( 1 - \frac{a^4}{b^4} \right) + p_2.$$

- 1.1 - §1.8.3 p. 43: The density of the fluid,  $\rho$  is never defined in problem 4. The last sentence of the problem should read: “If the plunger pushes a fluid of density  $\rho$  with a force  $F$ , what is the velocity at which fluid is emitted from the tip?”
- 1.1 - §1.9 p. 43: The coefficient of kinetic friction should be changed to 0.5 in the solution to problem 1 of the block problems. The only effect of this is to change the second-to-last sentence of the solution to read: “Now, friction contributes a force  $(0.5)(30\sqrt{2}) = 15\sqrt{2}$  N opposing the block’s motion, which means up the ramp in this case.”
- 1.1 - §1.9 p. 45: The second equation in the solution to problem 1 in the section on energy has incorrect units. The corrected line should read:

$$\implies \omega^2 = \frac{3}{5} \text{ rad}^2/\text{s}^2$$

- 1.0 - §1.9 p. 47: The solution to problem 2 in the momentum section has an integral that is evaluated incorrectly. The solution has been rewritten and now reads:

B - It is important to keep the notation straight in this problem. The  $r$  in the mass density  $\rho(r)$  refers to the distance between the origin and a point in the sphere, but when we compute the moment of inertia, the argument of the integral is the square

distance from *a point on the sphere to the axis of rotation*—in this case, the  $z$  axis. If we use  $s = r \sin \theta$  to denote this distance, then the moment of inertia is

$$\begin{aligned}
 I &= \int s^2 dm \\
 &= \int r^2 \sin^2 \theta \rho(r) dV \\
 &= \int_0^{2\pi} \int_0^\pi \int_0^R Ar^3 \sin^2 \theta r^2 \sin \theta dr d\theta d\phi \\
 &= \frac{1}{6} AR^6 \int_0^{2\pi} \int_0^\pi \sin^3 \theta d\theta d\phi \\
 &= \frac{\pi}{3} AR^6 \int_0^\pi \sin^3 \theta d\theta \\
 &= \frac{4\pi}{9} AR^6.
 \end{aligned}$$

- 1.0 - §1.9 p. 53: The last two equations in the solution to fluid mechanics problem 4 should contain  $v_2$  instead of  $v_1$ . The equations should read:

$$\frac{1}{2}\rho \left(v_2 \frac{a}{A}\right)^2 + \frac{F}{A} = \frac{1}{2}\rho v_2^2.$$

$$v_2^2 = \frac{2FA}{\rho(A^2 - a^2)}.$$

### 11.3 Electricity and Magnetism

- 1.1 - §2.1.2 p. 58: While the equation in the section on point charges in section 2.1.2 may be correct in cgs units, we are sticking to SI. It therefore needs an additional prefactor and should read:

$$V(x) = \frac{1}{4\pi\epsilon_0} \left( \frac{q}{|x-d|} - \frac{q}{|x+d|} \right).$$

- 1.0 - §2.1.2 p. 59: In the “Line charges and cylinders” section, the second and third equations should have factors of  $\epsilon_0$  in the denominator, and should read:

$$\begin{aligned}
 |\mathbf{E}| &= \frac{\lambda}{2\pi\epsilon_0 r} \\
 \mathbf{E} &= \frac{\lambda}{2\pi\epsilon_0 r} \hat{\mathbf{r}}.
 \end{aligned}$$

- 1.1 - §2.1.6 p. 64: The sentence before equation (2.17) should read: “The work required to put together  $n$  point charges is”.

- 1.1 - §2.1.6 p. 64: The sentence following equation (2.19) is missing a verb, and should read: “This simple formula can be extremely useful.”
- 1.0 - §2.1.8 p. 67: The statement of problem 3 should specify  $d \gg a$ , NOT  $d \ll a$ .
- 1.0 - §2.1.8 p. 67: Choices (A)-(D) of problem 4 should contain a factor of  $a$  in the denominator, NOT  $a^2$ .
- 1.1 - §2.2.3 p. 72: The sentence after equation (2.32) should specify the meaning of  $s$ :  $2\pi s$  is the circular distance around the center of the toroid.
- 1.1 - §2.2.4 p. 72: In the last sentence before equation (2.33), the word “using” should replace “use”.
- 1.0 - §2.2.4 p. 73: Equation (2.34) should read:

$$\mathbf{B}_2^{\parallel} - \mathbf{B}_1^{\parallel} = \mu_0 \mathbf{K} \times \hat{\mathbf{n}}.$$

- 1.1 - §2.2.7 p. 74: Choices (A), (B), (C), and (E) of magnetostatics problem 1 should contain a factor of  $a$  NOT  $a^2$  in the denominator.
- 1.1 - §2.2.7 p. 74: The separation between the two loops in problem 2 should be much *greater* than their radii, NOT much less.
- 1.1 - §2.3.3 p. 77: The expression for the emf in equation (2.46) is missing a minus sign and should read:

$$\mathcal{E} = -L \frac{dI}{dt}.$$

- 1.1 - §2.4.1 p. 80: The first sentence of section 2.4.1 should read “...located at...”.
- 1.1 - §2.4.4 p. 82: Question 1 should ask for the magnitude of the electric field, not the electric potential. The revised third sentence should read: “What is the magnitude of the electric field experienced by a test particle at  $(0, 0, 2z)$  from a dipole of moment  $2\mathbf{p}$ ?”
- 1.1 - §2.6.1 p. 84: The sentence beginning “The *physical* part...” is missing a verb and should read: “The *physical* part of the fields is just the real part.”
- 1.1 - §2.7.1 p. 88: This one is particularly embarrassing. The LHS of equations (2.71), (2.73), and (2.75) should be inverted ( $1/C_{eq}$ ,  $1/R_{eq}$ , and  $1/L_{eq}$  respectively).
- 1.1 - §2.7.2 p. 89: In section 2.7.2, Kirchoff should be spelled with two h’s.
- 1.0 - §2.7.4 p. 89: The equation above (2.82) has misplaced parentheses. It should read:

$$I = \frac{V}{R} \left( 1 - \exp \left( -\frac{R}{L} t \right) \right).$$

- 1.0 - §2.8 p. 91: The equation for work in the solution to the electrostatics problem 4 should read:

$$W = \frac{6q^2}{4\pi\epsilon_0 a}.$$

- 1.1 - §2.8 p. 92: The solution to magnetostatics problem 1 should have an extra factor of  $a$  in the numerator, and the magnitude of the magnetic field should be taken everywhere. The answer should therefore read:

$$\frac{\mu_0 I}{4a}.$$

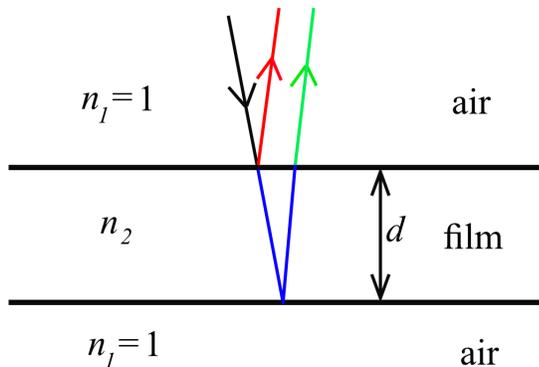
- 1.0 - §2.8 p. 95: The last sentence of the solution to dipoles problem 1 should read: “The electric field along the  $z$ -direction is simply the  $z$ -derivative of the potential  $V$ , so the electric field will scale as  $p/z^3$  and will be reduced by a factor of 4.” The solution to this problem should be choice (A) instead of (B).

## 11.4 Optics and Waves

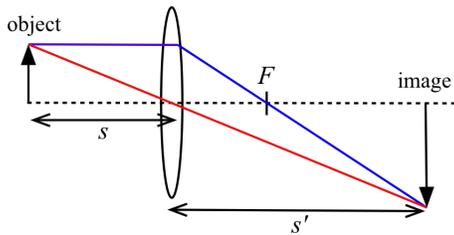
- 1.1 - §3.1.2 p. 101: In the second-to-last sentence of section 3.1.2  $\mathbf{k} \cdot \mathbf{r}$  should replace  $\mathbf{k} \cdot \hat{\mathbf{r}}$ .
- 1.1 - §3.1.4 p. 102: Equation (3.5) should read:

$$\omega/k = c/n \quad (\text{for light waves}).$$

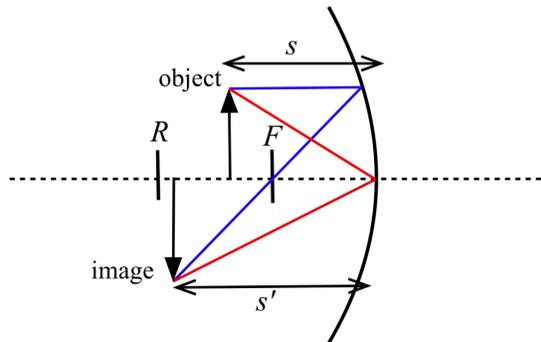
- 1.1 - §3.2.4 p. 108: In the printed copy, the figure has no color. Here’s the figure with color:



- 1.1 - §3.3.2 p. 110: In the printed copy, the figure has no color. Here’s the figure with color:



(a) A converging lens



(b) A concave mirror

- 1.1 - §3.5 p. 115: In problem 8, the order of interference extrema is not stated correctly. The question should read: “If the same light is directed at a different double-slit arrangement with slit separation  $d'$ , the position of the third interference minimum corresponds to the position of the old second interference *maximum* (after the central maximum). What is  $d'$  in terms of  $d$  and  $\lambda$ ?”
- 1.1 - §3.6 p. 119: In the solution to optics problem 11,  $\beta = \pi/2 - \alpha$  NOT  $\beta = \pi - \alpha$ . The rest of the solution is correct and this equation is just misprinted.

## 11.5 Thermodynamics and Statistical Mechanics

- 1.1 - §4.1.2 p. 124: Equation (4.9) should read  $S = Nk_B \log VT^{3/2}/N + \text{constants}$ .
- 1.1 - §4.1.3 p. 124: In the sentence before the first equation in section 4.1.3  $n$  should be replaced by  $N$ .
- 1.0 - §4.1.3 p. 123: Equation (4.12) contains the correct number of degrees of freedom, but not the correct types of degrees of freedom. There should be only two rotational degrees of freedom: one per axis perpendicular to the axis connecting the two atoms. There should also be two vibrational degrees of freedom: one for the kinetic energy of the vibration of the two atoms, and another for the potential energy between the two atoms. The corrected version of equation 4.12 should read:

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{4}ms^2 + \frac{1}{2}ks^2.$$

- 1.1 - §4.1.3 p. 125: In the third sentence after equation (4.11), the proportionality should read  $E \propto 1/(2\beta)$  NOT  $E \propto 1/1(2\beta)$ .
- 1.0 - §4.2.3 p. 126: Equation (4.20) should be taken at constant volume rather than constant pressure, and therefore should read:

$$T = \left( \frac{\partial U}{\partial S} \right) \Big|_V.$$

- 1.1 - §4.2.4 p. 129: The section on reversible processes erroneously states that a reversible process must have zero change in entropy. In fact, the entropy of a system undergoing a reversible process can change, but the combined entropy of a system and its surroundings must remain constant.
- 1.1 - §4.2.6 p. 133: The first equation in the section on monatomic ideal gases has an inconsistent use of the proportionality sign. The first proportionality is correct since the normalizing factor of  $h^{-3}$  is omitted from the integral. However, the factor of  $h^{-3}$  should not appear in the last part of the equation. The revised equation should read:

$$Z \propto \int e^{-\beta \mathbf{p}^2/2m} d^3p d^3x = V (2\pi mkT)^{3/2}.$$

- 1.1 - §4.2.6 p. 133: The equation in the first sentence after equation (4.40) should read:  $c_V = (3/2)Nk_B$ . Instead of  $c_V$ , the equation erroneously used  $c_P$ .
- 1.1 - §4.4 p. 136: Problem 3 should specify the heat capacity is at constant volume.
- 1.1 - §4.5 p. 139: The answer to problem 3 is A NOT B. The solution should be corrected to read: “A - A diatomic gas in two dimensions has five quadratic degrees of freedom: three translational (two for the center of mass and one for the atomic separation), one rotational (the angle with respect to a fixed axis), and one vibrational (a harmonic oscillator potential for the atomic separation). This gives 5 quadratic degrees of freedom in total, so by the equipartition theorem, choice A is correct.”
- 1.1 - §4.5 p. 140: In the solution to problem 5, the initial entropy should be  $S_1 = k_B \log 3$  NOT  $S_1 = k_B \log 2$  since the degeneracy of the  $n = 1$  state is 3, as stated in the solution. This misprint is propagated through the rest of the solution. The correct answer should be  $\Delta S = k_B \log 2$  NOT  $\Delta S = k_B \log 3$ , or choice (D) NOT choice (E).
- 1.1 - §4.5 p. 140: In the solution to problem 5, the expression for the energies of the 3D harmonic oscillator in the first sentence should read:  $E_n = \hbar\omega(n_x + n_y + n_z + 3/2)$ .

## 11.6 Quantum Mechanics and Atomic Physics

- 1.1 - §5.1.2 p. 149: The third sentence of the third bullet point should read “...if a particle is in a stationary state  $\psi_n$  which is taken to be real, the expectation value of its momentum *must* vanish.”
- 1.1 - §5.1.2 p. 149: The wavefunction in the last sentence of the third bullet point should read:  $\Psi(x, t) = \psi_1 e^{-iE_1 t} + \psi_2 e^{-iE_2 t}$ .
- 1.1 - §5.2.2 p. 158: In equation (5.28) in section 5.2.2, the expression for the energies of the 3D harmonic oscillator should read

$$E_N = \left( N + \frac{3}{2} \right) \hbar\omega \text{ with } N = n_1 + n_2 + n_3.$$

- 1.1 - §5.3.5 p. 164: The first equation in section 5.3.5 should contain factors of  $i$  in the exponentials. It should read:

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx}, & x \leq -a \\ \text{something}, & -a \leq x \leq a \\ Ce^{ikx}, & x \geq a \end{cases}$$

- 1.1 - §5.4.2 p. 169: The second bullet point of the list at the end of section 5.4.2 should read “ $\phi$  dependence” NOT “ $m$  dependence.”
- 1.1 - §5.5.1 p. 175: Equations (5.53) and (5.54) are missing factors of  $\hbar$  and should read:

$$\hat{S}_+ |\uparrow\rangle = 0, \quad \hat{S}_- |\uparrow\rangle = \hbar |\downarrow\rangle.$$

and

$$\hat{S}_+ |\downarrow\rangle = \hbar |\uparrow\rangle, \quad \hat{S}_- |\downarrow\rangle = 0.$$

- 1.1 - §5.6.1 p. 182: In second bullet of the bulleted list of section 5.6.1, the phrase “large and positive” in the second sentence should be changed to “small and positive”.
- 1.1 - §5.7.6 p. 189: Equation (5.62) is missing its Bose-Einstein factor. It should read:

$$I(\omega) \propto \frac{h\omega^3}{c^2} \frac{1}{e^{h\omega/k_B T} - 1}.$$

- 1.1 - §5.8 p. 196: In the solution to problem 5 of the section on “Other Standard Hamiltonians,” the expression for  $k$  should read:

$$\frac{\hbar^2 k^2}{2m} + V_0 = E \implies k = \frac{\sqrt{2m(E - V_0)}}{\hbar}.$$

## 11.7 Special Relativity

- 1.1 - §6.6 p. 215: In problem 3 of the special relativity section, the particle should decay into three particles of equal energy. The revised statement of the problem should read: “A particle of mass  $M$  and energy  $E$  decays into three identical particles of equal energy. What is the magnitude of the momentum of one of the decay products of mass  $m$ ?”
- 1.1 - §6.6 p. 217: In problem 9 of the special relativity section, both spaceships are traveling along parallel trajectories and all dimensions should be measured parallel to the axis of their trajectories.
- 1.0 - §6.7 p. 214: The last sentence of the solution to problem 3 should read “...indeed,  $|p|$  goes imaginary when  $E < 3mc^2$ .”

- 1.1 - §6.7 p. 218: In the solution to problem 3 of the special relativity section, the words “By symmetry” should be deleted.
- 1.1 - §6.7p. 220: The Taylor expansion in the solution to problem 10 is missing a factor of 2 in the denominator of the second term. The rest of the solution is correct. The corrected line should read:

$$T = \left( \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} - 1 \right) mc^2 = \left( 1 + \frac{v^2}{2c^2} + \frac{(-1/2)(-3/2)v^4}{2!c^4} + \dots - 1 \right) mc^2$$

## 11.8 Laboratory Methods

- 1.1 - §7.1.2 p. 222: In the third bullet point in section 7.1.2, the equation should read  $y = C \cdot 10^{bx}$ , NOT  $y = C \cdot 10^x$ .
- 1.1 - §7.2.1 p. 223: The “statement” in section 7.2.1 under the uncertainty bullet point should be revised to read: “this measurement has an uncertainty of 10%”.
- 1.1 - §7.4 p. 229: The second sentence of the second paragraph in section 7.4 should be revised to read: “The surface area of the ball is  $4\pi R^2$ , but the surface that the bullets “see” is the area of the shadow cast by the ball,  $\pi R^2$ .”
- 1.1 - §7.7 p. 239: In the solution to problem 9,  $LC$  should include the units  $s^2$ .

## 11.9 Specialized Topics

- 1.1 - §8.2.1 p. 246: the discussion of crystal lattices in this section did not distinguish between the primitive unit cell and the conventional unit cell. The two paragraphs following the diagram should be replaced by:

The key here is *repetition*: the crystal structure is infinite, so we have to define a repeating pattern, known as a *unit cell*. The cubes drawn above are known as *conventional unit cells*, and they make the cubic symmetry apparent by containing a cube with atoms at every vertex, but some add extra points to it: the body-centered cubic contains an atom at the center, and the face-centered cubic contains atoms at the centers of each of the faces. To construct the rest of the crystal, you can tessellate space in all three dimensions using these unit cells.

Despite making the symmetry manifest, it may be the case that the conventional unit cell is not the smallest repeating pattern, where smallest means “containing the least number of atoms.” This smallest pattern is known as the *primitive unit cell*. This is the case for the cubics: the primitive unit cell for the BCC is an octahedron with half the volume of the conventional cell, and the primitive unit cell for the FCC is a parallelepiped with one quarter the volume of the conventional cell. Similarly, the volumes of the conventional unit cells are all equal, but the interatomic distances are all different: for a cube of side  $a$ , the simple cubic has distance  $a$ , the BCC has distance

$a\sqrt{3}/2$ , and the FCC has distance  $a\sqrt{2}/2$ . A favorite GRE question type gives you the volume of a primitive unit cell and asks for the interatomic distance or the volume of the conventional unit cell. By the way, these lattice structures are examples of *Bravais lattices*: there are fourteen of them, but the only ones which show up on the GRE with any frequency are the three cubic types given above.

## 11.10 Special Tips and Tricks for the Physics GRE

- 1.1 - §9.4 p. 263: In the third paragraph of section 9.4, the first sentence should read “... because the electron’s binding energy is much less than its mass...” NOT “... because the electron energy is much less than its binding energy”.
- 1.1 - §9.6 p. 267-269: Section 9.6 contains several errors and has been heavily revised. The revised version is here:

How many piano tuners are there in New York City? This is a classic estimation problem attributed to Enrico Fermi, and it’s interesting because it requires more than just a single “educated guess”: a good solution will invoke several order-of-magnitude approximations and combine them together using appropriate formulas. For instance, you might start by estimating the population of New York City, then estimating the size of the average family to get an approximate number of families, then multiply by a factor representing the proportion of families that have a piano, and so on. Whole Fermi problems as defined above are unlikely to show up on the GRE, simply because they often require than 2-3 minutes to carry out all the approximations and arithmetic. However, they are an *excellent* way to review your knowledge of important formulas by applying them to a real-world problem. Furthermore, an intermediate calculation in a Fermi problem (maybe involving only one approximation and one formula) is a classic GRE question – see Section 9.5 for an example using intermolecular spacing.

OK, that’s enough talking: let’s get to an example.

*What is the number flux of solar visible-spectrum photons arriving at Earth’s atmosphere?*

In layman’s terms, how bright does the sun appear? (You might want to test your intuition by making an order-of-magnitude guess at that number before continuing.) This is an excellent Fermi problem because it brings in so many different formulas and concepts from several different areas of physics, plus a good helping of geometry and back-of-the-envelope estimating. Here’s how we could approach this problem:

1. Assume the sun is a black-body whose spectrum peaks in the visible range, find its temperature using Wien’s law, then find the total power from the Stefan-Boltzmann law. Using the shape of the black-body spectrum, convert power to photon flux.

- Use the average Earth-sun distance to find the photon flux on a sphere at the radius of Earth's orbit. Then find the portion of this sphere subtended by the sun-facing surface of the Earth, since only photons in this solid angle will hit the Earth.

Of course, this probably isn't the only way to approach this problem, but it's pretty straightforward and affords good practice with this style of information. Along the way we'll compare our estimated values with the true values and see how close we come at the end.

**Step 1:** We can justify the approximation about the sun's spectrum just by looking outside: the sun appears bright and mostly white, which means that our eyes receive a large number of photons from all over the visible spectrum. Since the Planck distribution, which is proportional to  $\frac{\omega^3}{e^{h\omega/kT} - 1}$ , falls off rather sharply on either side of the maximum, if the maximum were outside the visible spectrum, the sun would appear either very red or very blue. So we can safely assume the spectral maximum to be at the center of the visible spectrum, approximately 500 nm. (The visible spectrum is about 300-800 nm, a range you should be familiar with.) Using Wien's displacement law  $\lambda_{max} \approx 3 \times 10^{-3} \text{m} \cdot \text{K}/T$ , we find the surface temperature of the sun is about 6000 K. The actual temperature is 5778 K, with nearly all the difference coming from using a more accurate value for Wien's constant – not bad so far!

Now, the Stefan-Boltzmann law is  $P/A = \sigma T^4$ , with  $\sigma \approx 6 \times 10^{-8} \text{W m}^{-2} \text{K}^{-4}$  (you'd have to look that one up, since it's not given on the GRE formula sheet) the Stefan-Boltzmann constant. This actually gives the power per unit surface area, as you can see from the units. We're left with

$$\frac{P}{A} \approx 8 \times 10^6 \text{ W m}^{-2}.$$

As we've noted, the black-body distribution is peaked sharply at the maximum, so we can crudely assume that *all* photons have wavelength 500 nm, with energy

$$E_\gamma = hc/\lambda \approx 4 \times 10^{-19} \text{ J}.$$

Since power has units of energy per time, we can use this photon energy to convert from energy to photon number, giving a photon rate per sun surface area of

$$\frac{dN}{dt dA_{sun}} = 8 \times 10^6 \text{ J s}^{-1} \text{m}^{-2} \times \frac{1 \text{ photon}}{4 \times 10^{-19} \text{ J}} \approx 2 \times 10^{25} \text{ m}^{-2} \text{ photon/s}.$$

**Step 2:** We're now going to make some quick-and-dirty approximations. We'll need the fact that the Earth-sun distance is 8 light-minutes (a good fact to memorize, if not for the GRE then for general physics knowledge), which works out to

$$d_{Earth-sun} = 8 \text{ min} \times 3 \times 10^8 \text{ m/s} = 1 \times 10^{11} \text{ m}.$$

We also need the radius of the Earth, which we can estimate as follows. The coast-to-coast distance of the United States is about 3000 miles or about 5000 km, and one can imagine fitting 10 US-sized countries around the equator (certainly more than 1, and certainly less than 100), so the circumference of the earth is about  $C_{Earth} = 5 \times 10^7$  m, giving a radius of  $r_{Earth} = C_{Earth}/2\pi \approx 1 \times 10^7$  m. Now, the radius of the sun is somewhere between these two numbers, so a reasonable guess would be  $r_{sun} = 1 \times 10^9$  m. These guesses are actually surprisingly close to the true values:

$$\begin{aligned}d_{Earth-sun} &= 1.496 \times 10^{11} \text{ m,} \\r_{Earth} &= 6.37 \times 10^6 \text{ m,} \\r_{sun} &= 6.96 \times 10^8 \text{ m.}\end{aligned}$$

Now we can multiply by  $4\pi r_{sun}^2$  to get the total photon flux at the surface of the sun:

$$\frac{dN}{dt} \approx (2 \times 10^{25} \text{ m}^{-2} \text{ photon/s})(4\pi \times 10^{18} \text{ m}^2) \approx 2 \times 10^{44} \text{ photon/s.}$$

This is still not quite what we want – it’s the flux at a sphere of radius equal to the Earth-sun distance, but the Earth only covers a tiny portion of this sphere. The surface area of the sphere is  $4\pi d_{Earth-sun}^2$ , and since the Earth is very small, it cuts out an area approximately equal to its cross-sectional area,  $\pi r_{Earth}^2$ . Multiplying by the ratio of these areas, we have finally

$$\frac{dN_{earth}}{dt} = 2 \times 10^{44} \text{ photon/s} \times \frac{\pi r_{Earth}^2}{4\pi d_{Earth-sun}^2} \approx \boxed{1 \times 10^{35} \text{ photon/s}}.$$

Now, we dropped *lots* of factors of 2 and such along the way, so we shouldn’t be too surprised to be off by an order of magnitude or two. But this number is fairly important for biology, as it controls things like the growth rate of plants due to photosynthesis, so we can look up the actual value:<sup>1</sup>

$$\begin{aligned}\frac{dN_{total}}{dt} &\approx 1.02 \times 10^{36} \text{ photon/s} \\ \frac{dN_{visible}}{dt} &\approx 3.06 \times 10^{35} \text{ photon/s.}\end{aligned}$$

Not bad! The first thing to note is that the difference between the *total* photon flux and the *visible spectrum* photon flux is only a factor of 3 or so, which tells us that our approximation of the entire spectrum consisting of visible photons was perfectly fine for a rough estimate. Overall, we were off by about 1 order of magnitude, but this is not too surprising: we used inaccurate values for various distances, and since they show up squared in many formulas, these errors magnify. Furthermore, this is the

---

<sup>1</sup>Taken from [www.bionumbers.org](http://www.bionumbers.org), converting flux per area into total flux by multiplying by  $2\pi r_{Earth}^2$  for the sun-facing surface.

flux at the surface of the Earth, not at the atmosphere, and so ignores the effects of dipole radiation from molecules in the atmosphere that could in principle magnify the number of photons arriving on the ground. But considering how rough we were, being within a factor of 10 is astonishingly good!

### 11.11 Exam 1

- 1.2.0 (b:1.1) - p. 272: The units of problem 1 are potentially ambiguous. We intend for  $\sqrt{g} = \sqrt{9.8}$  with no units.
- 1.2.0 (b:1.1) - p. 275: The wording of problem 15 of exam 1 should be changed from “...a distance  $d$  above an infinite conducting plate” to “...a distance  $d$  from an infinite conducting plate” to emphasize that the effect of gravity should be neglected. The parenthetical comment should also be changed to “(Ignore gravity and all relativistic effects)”.
- 1.2.0 (b:1.1) - p. 290: In problem 85, choice (C) should not include the words “the selection rule”. It should simply read: “It violates  $\Delta m = \pm 1$  or 0.”
- 1.2.0 (b:1.1) - p. 369: The last sentence of the solution to problem 78 should read: “This is just  $0.1 = \exp(-\lambda T)$ , or  $\lambda = -(1/T) \log 0.1$ .”
- 1.2.0 (b:1.1) - p. 371: To maintain consistent notation, the equation in the solution to problem 88 should read:

$$b_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin nx \, dx = \frac{1}{\pi} (-\pi \cos(\pi) - \pi \cos(-\pi)) = 2.$$

### 11.12 Exam 2

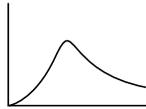
- 2.2.0 (b:1.1) - p. 293: Problem 5 is ambiguous and will be replaced in edition 2.0.
- 2.2.0 (b:1.1) - p. 306: In problem 42, “muonic hydrogen” should be replaced by “muonium”.
- 2.2.0 (b:1.1) - p. 308: In problem 53, choices (B) and (D) should be multiplied by a factor of -1.
- 2.2.0 (b:1.1) - p. 313: In the statement of problem 70, the object should be placed 12.5 cm to the left of A (in agreement with the figure), NOT 12 cm.
- b:2.0 - p. 380: The solution of problem 32 should read: “Since  $Q = 0$  for a reversible adiabatic process, there is no change in entropy.” The answer choice is still correct.
- 2.2.0 (b:1.1) - p. 382: In the solution of problem 42, “muonic hydrogen” should be replaced by “muonium”.

- 2.2.0 (b:1.1) - p. 384: In the solution to problem 53, the equation should read:

$$\mathbf{E} = -\frac{q}{4\pi\epsilon_0} \frac{1}{(L/\sqrt{3})^2} \hat{\mathbf{y}} = -\frac{3q}{4\pi\epsilon_0 L^2} \hat{\mathbf{y}}.$$

### 11.13 Exam 3

- 3.2.0 (b:1.1) - p. 324: Problem 7 should be reworded to read: “What is the difference in energy between the  $n = 5$  and the  $n = 1$  states of the 1D quantized harmonic oscillator?”
- 3.2.0 (b:1.1) - p. 325: The notation of Problem 20 is potentially confusing, since  $B$  does not have units of magnetic field. Instead, the second sentence should read “There is a magnetic field, pointing into the page, of magnitude  $B = Cx$ , where  $x = 0$  is the initial position of the bar.” Correspondingly, all answer choices should have the letter  $B$  replaced by  $C$ .
- 3.2.0 (b:1.1) - p. 329: In problem 26, the second sentence of the problem should be reworded to read: “Which could be the potential as a function of radius?”. Additionally, choice (E) should be changed to:



- 3.2.0 (b:1.1) - p. 330: In problem 30, the point charge in the figure should have a charge of  $Q$ , NOT  $q$ .
- 3.2.0 (b:1.1) - p. 333: In problem 33, the system should be taken to be spin-0.
- 3.2.0 (b:1.1) - p. 335: In problem 54, the “Note” is missing a minus sign in the exponent inside the integral. It should read: “Note that

$$\int_{-\infty}^{\infty} e^{-x^2/c^2} = c\sqrt{\pi}.”$$

- 3.2.0 (b:1.1) - p. 397: In the solution to problem 6, the second sentence should begin with: “The next *smallest* is the Lamb shift...”
- 3.2.0 (b:1.1) - p. 397: In the solution to problem 8, the expression for the centrifugal force should contain a factor of  $\cos(45^\circ + \theta)$  NOT  $\cos(45^\circ - \theta)$ . This error does not affect the final result.
- 3.2.0 (b:1.1) - p. 413: The answer to problem 72 is given in seconds, while the answer choices are given in eV. The answer should be quoted in eV and is  $\Delta E = 1.1 \times 10^{-6}$  eV.